Trade-based Asset Model for Combinatorial Prediction Markets

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- Combinatorial Prediction Markets
  - Bayesian Networks (BN) represents the joint distribution in factorized form
  - Logarithmic market scoring rule to update user's asset

$$a_{\mathbf{v}}^{u} + b \log \frac{x(t|\mathbf{H}=\mathbf{h})}{p(t|\mathbf{H}=\mathbf{h})}$$

  - TASK-1 (market): Probability update for maintaining market distribution
  - Solution: junction tree inference
  - TASK-2 (user): asset management for calculating (1) expected asset; (2) minimum asset, a.k.a. cash
  - Solution: trade-based asset model

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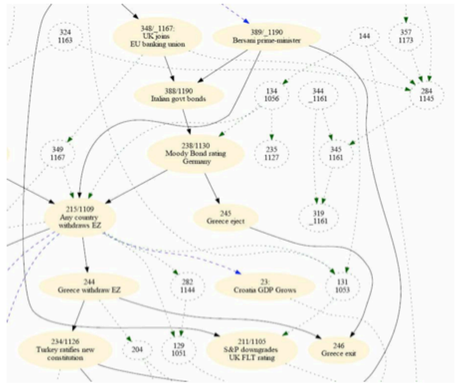
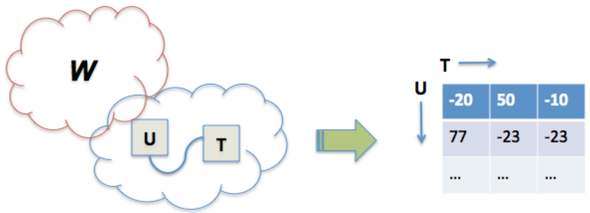
BN-based Prediction Markets



Basic Unit – Trade Asset Block



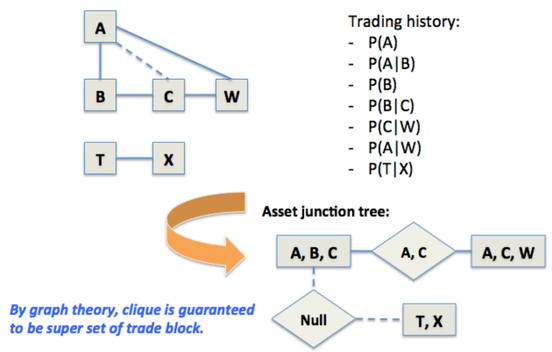
p_i is current probability, x_i is the edit by user

$$\begin{aligned} x_i &= x(\mathbf{W}|T,\mathbf{U})x(T|\mathbf{U}=\mathbf{u})x(\mathbf{U}=\mathbf{u}) \\ &= p(\mathbf{W}|T,\mathbf{U})x(T|\mathbf{U}=\mathbf{u})p(\mathbf{U}=\mathbf{u}) \end{aligned} \Rightarrow \frac{x_i}{p_i} = \frac{x(T|\mathbf{U}=\mathbf{u})}{p(T|\mathbf{U}=\mathbf{u})}$$

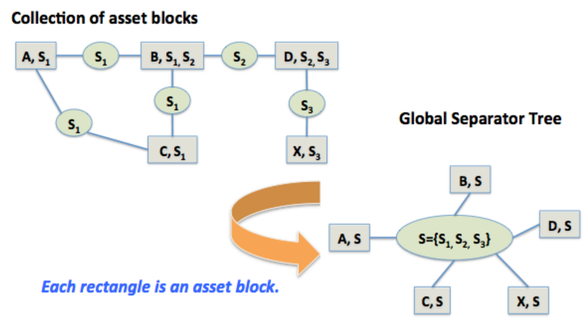
Therefore:

$$a_i^1 = a_i^0 + b\log(\frac{x_i}{p_i}) = a_i^0 + b\log(\frac{x(T|\mathbf{U}=\mathbf{u})}{p(T|\mathbf{U}=\mathbf{u})}) \prec b\log(\frac{x(T|\mathbf{U}=\mathbf{u})}{p(T|\mathbf{U}=\mathbf{u})})$$

DAC (Dynamic Asset Cluster) Example



Lazy Solution: Using Global Separator



Lazy evaluation means separator isn't built in memory.

Complexity Analysis

Name	Meaning	Design Values
n	NoV involved in all open trades	10—100
k	number of asset blocks	$\sqrt{n}$
m	NoC in the *DAC* junction tree	$k/2$
v	NoV in the biggest asset block	8
s	NoV in the biggest clique	11
r	number of states per variable	3

- DAC

$$\mathcal{O}(kv^2 + n\log(n) + m^2 + kv + kr^s) \sim \mathcal{O}(1200 + 10 \times 3^s)$$

- Global Separator

$$\mathcal{O}(k \times v) + \mathcal{O}(k \times r^{z+v})$$

Where z is number of variables in global separator.

Experimental Setup

- Chained trades
  - Bounded clique size of 2 variables for DAC
  - Global Separator of n-2 variables
- 5 cliques with sparse trades
  - Fixed size of cliques and global separator
  - Sparse trades in cliques
  - Number of entries matter

Results